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# Drift kinetic Alfvén wave in the presence of inhomogeneous magnetic field

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**Abstract** Particle aspect analysis is used to evaluate dispersion relation, field aligned current, perpendicular current (perpendicular to $B_0$) and growth rate of kinetic Alfvén wave propagating in the inhomogeneous magnetospheric plasma. The effect of magnetic field inhomogeneity is examined in an anisotropic plasma incorporating the effect of finite Larmor radius corrections. The applicability of the investigation is discussed for auroral phenomena at the substorm times.

**Keywords** Kinetic Alfven wave, magnetosphere, inhomogeneity

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## 1. Introduction

The kinetic Alfvén wave (KAW) is the Alfvén wave for which wave particle interactions are important [1–4]. The wavelength of the kinetic Alfvén wave ($k_{\parallel}/k_{\perp} << 1$) is theoretically scaled by $2\pi\rho_i$ where $\rho_i$ is the typical ion gyroradius. This wave has gained much attention recently in connection with particle acceleration along the auroral field lines [5–7]. The KAW can also be an active agent to heat the plasma in the solar corona [8]. Moreover, the structure of the auroral arcs seems to be determined partly by this wave [9], as the latitudinal scale of the arcs is comparable to the ion gyroradius.

Taking into account the finite values of the Larmor ion radius or the electron inertia in the same frequency region, there also exists a much shorter scale mode with components of the wave vector $k_{\perp} >> k_{\parallel}$. These waves were called kinetic Alfvén waves by Hasegawa and Chen [3]. Though KAW retain the main properties of magnetohydrodynamic (MHD) Alfvén waves, they have some important new properties as well, including (i) a dependence of the wave dispersion on the wave vector component, and (ii) the presence of a non-zero component of the electric field. Owing to these properties, KAWs can interact with plasma particles and other kinds of waves more effectively than MHD Alfvén waves [10].

Recent observations from the Freja satellite [11,12], indicate that the auroral plasma can evolve to a highly coherent electromagnetic structure. They are interpreted as kinetic Alfvén waves. KAWs have been invoked in association with auroral current and particle acceleration since the pioneer work of Hasegawa and Mima [13], Goertz and Bosewell [14], Wu *et al* [15,16] and Wang *et al* [17]. Theoretical studies of both high latitude ULF phenomena and the waves at equatorial to middle latitudes involve kinetic Alfvén waves. These waves are considered in a number of models as an agent to magnetosphere-ionosphere coupling. However, the finite Larmor radius of warm ions in the equatorial magnetosphere can determine the perpendicular wavelength of kinetic Alfvén waves [18]. Even if a formal distinction between the external KAW and the internal drift character is helpful to understand locally how the particles interact with the waves, one has to realize that the kinetic Alfvén wave, the ion-acoustic and the drift waves are here strongly coupled [19].

Field-aligned currents play a dominant role in the study of magnetized plasmas of magnetosphere-ionosphere coupling. In the magnetohydrodynamic description (valid where time and spatial scales of motion are large compared to the gyroperiod and gyroradius, respectively), if perturbations of flow develop on one part of the flux tube, it is communicated to entire flux tube. They are perhaps of most importance in magnetospheric physics in the study of coupling between regions where different dynamical conditions prevail but which are threaded by the same field [20]. Under this situation kinetic Alfvén waves may be generated by the density inhomogeneity of a plasma sheet and propagate to the ionosphere.

In the context of magnetosphere-ionosphere coupling, kinetic Alfvén waves have been studied in both homogeneous and inhomogeneous magnetic fields in various investigations using basically, a magnetohydrodynamic approach [6,21–23]. Leonovich and Mazur [22,23] have studied kinetic Alfvén waves in a two dimensional inhomogeneous plasma in a curved magnetic field. In the past, the development of kinetic Alfvén wave theory was limited to homogeneous magnetic fields, however, in the magnetospheric structures due to converging magnetic field lines at the poles and gradient of magnetic field in the plasma sheet the treatment of the wave in homogeneous fields may be inappropriate. In this view the magnetic field inhomogeneity has been taken into consideration to explain the auroral electrodynamics by kinetic Alfvén wave in the present analysis.

In the recent past, Alfvén waves and kinetic Alfvén waves are analyzed using particle aspect analysis and the dispersion relation currents, growth-rate are investigated for the homogeneous magnetic field [24–27]. Our purpose in this paper is to investigate the effect of inhomogeneous magnetic field on the kinetic Alfvén waves in the inhomogeneous magnetospheric plasma. An alternative model usually called a particle aspect analysis [28–30] is applied which offers an advantage over the magnetohydrodynamic approach in dealing with the finite Larmor radius effect and temperature anisotropy in the inhomogeneous magnetoplasma alongwith evaluation of currents and energy transferred in the wave-particle interaction mechanism. The basic assumptions are those of earlier work on the kinetic Alfvén wave [26] in which the plasma has been considered to consist of resonant and non-resonant particles and the wave growth was discussed by the energy conservation method.

We have considered a kinetic Alfvén wave propagating obliquely to the constant magnetic field ($z$-direction), and two different potentials in the $x$-$z$ plane for the evaluation of the charged particle trajectory. The direction of the density gradient is along the $y$-axis. The inhomogeneity of magnetic field lies along the $x$-direction.

The organization of this paper is as follows In Section 2, we evaluate the trajectories of the charged particles and in Section 3, the density perturbation is considered. Section 4 considers the dispersion relation. In Section 5, we evaluate the current densities and, in Section 6, the energy balance and growth rate are considered. Results and discussion are presented in Section 7.

## 2. Basic Trajectories

In the mathematical analysis, we follow, the procedures considered by Terashima [31], Tiwari and Varma [28], Varma and Tiwari [30], Tiwari *et al* [32] and Baronia and Tiwari [26]. The kinetic Alfvén wave is assumed to start at $t = 0$ when the resonant particles are undisturbed. We are interested in the kinetic Alfvén waves which satisfy the condition

$$V_{T\|i} \ll \frac{\omega}{k_\|} \ll V_{T\|e},\ \omega \ll \Omega_i, \Omega_e : k_\perp^2 \rho_e^2 \ll k_\perp^2 \rho_i^2 < 1, \tag{1}$$

where $V_{T\|i}$ and $V_{T\|e}$ are the mean velocities of ions and electrons along the magnetic field, $\Omega_{i,e}$ are gyration frequencies and $\rho_{i,e}$ the mean gyroradii of the respective species. $k_\perp$ and $k_\|$ are the components of wave vector $\bar{k}$ perpendicular and parallel to the magnetic field.

We begin with the two potential representation of wave electric field of the form [26,27,33]

$$E_\perp = -\nabla_\perp \phi$$

and $$E_\| = -\nabla_\| \psi$$

to decouple the compressional mode and the wave electric field.

$$\bar{E} = E_\perp + E_\|,$$
$$\phi = \phi_1 \cos(k_\perp x + k_\| z - \omega t),$$
$$\psi = \psi_1 \cos(k_\perp x + k_\| z - \omega t), \tag{2}$$

where $\phi_1$ and $\psi_1$ are assumed to be a slowly varying function of time $t$, and $\omega$ is the wave frequency.

The equation of motion of particle is :

$$m\frac{d\bar{v}}{dt} = q\left(\bar{E} + \frac{1}{c}\bar{v} \times \bar{B}\right), \tag{3}$$

where the collision between particles are neglected. The valocity $\bar{v}$ can be expressed as a sum of the unperturbed valocity $\bar{V}$ and the perturbed velocity $\bar{u}$, *i.e.* $\bar{v} = \bar{V} + \bar{u}$, $\bar{u}$ is determined by following set of equations [26,27]

$$\frac{du_+}{dt} + i\Omega u_+ = \frac{q}{m}\left[\phi_1 k_\perp - \frac{V_\| k_\perp k_\|}{\omega}(\phi_1 - \psi_1)\right]$$
$$\times \sin(k_\perp x + k_\| z - \omega t),$$
$$\frac{du_\|}{dt} = \frac{q}{m}\left[\psi_1 k_\| + \frac{V_\perp k_\perp k_\|}{\omega}(\theta_1 - \psi_1)\cos(\Omega t - \theta)\right]$$
$$\times \sin(k_\perp x + k_\| z - \omega t), \tag{4}$$

where $u_+ = u_x + iu_y$, $\theta$ is the initial phase of velocity, $\Omega = qB_0/mc$, $u_x$ and $u_y$ are the perturbed velocities in the $x$ and $y$ directions respectively.

The slowly varying quantities $\phi_1$ and $\psi_1$ are treated as constants.

We start by taking the trajectories of free gyration as [28,32,34]

$$x(t) = x_0 + \frac{V_\perp}{\Omega}[\sin(\Omega t - \theta) + \sin\theta]$$
$$+ \frac{\epsilon_B V_\perp^2}{2\Omega^2}\left\{\frac{1}{2}[-\cos(2\Omega t - 2\theta) + \cos 2\theta]\right.$$
$$\left. + 2(\cos\Omega t - 1)\cos^2\theta + \sin\Omega t \sin 2\theta\right\},$$

$$y(t) = y_0 + \frac{V_\perp}{\Omega}[\cos(\Omega t - \theta) + \cos\theta]$$

$$+ \frac{\epsilon_B V_\perp^2}{2\Omega^2}\Big\{\Omega t + \frac{1}{2}[\sin(2\Omega t - 2\theta) + \sin 2\theta]$$

$$+ (\cos\Omega t - 1)\sin 2\theta - 2\sin\Omega t \cos^2\theta\Big\},$$

$$z(t) = z_0 + V_\parallel t, \tag{5}$$

where $\epsilon_B = B^{-1}\, dB/dx$ is the inverse scale length of the magnetic field gradient, $\bar{r}_0 = (x_0, y_0, z_0)$ is the initial position of the particles at $t = 0$, where the wave is assumed to start. Eq. (4) is solved by replacing the coordinates of charged particles to that of free gyration [eq. (5)], which provides the perturbed velocity $\bar{u}(t)$ and can be further transformed to $\bar{u}(\bar{r}, t)$ by the use of eq. (5) once again.

Thus, $$u_x(\bar{r}, t) = -\frac{q}{m}\left[\phi_1 k_\perp - \frac{V_\parallel k_\perp k_\parallel}{\omega}(\phi_1 - \psi_1)\right]A_1 A_2$$

$$\times\left[\frac{\Lambda_n}{a_n^2}\cos\xi_{nl} - \frac{\delta}{2\Lambda_{n+1}}\cos(\xi_{nl} - \Lambda_{n+1}t)\right.$$

$$\left. - \frac{\delta}{2\Lambda_{n-1}}\cos(\xi_{nl} - \Lambda_{n-1}t)\right],$$

$$u_y(\bar{r}, t) = -\frac{q}{m}\left[\phi_1 k_\perp - \frac{V_\parallel k_\perp k_\parallel}{\omega}(\phi_1 - \psi_1)\right]A_1 A_2$$

$$\times\left[-\frac{\Omega}{a_n^2}\sin\xi_{nl} - \frac{\delta}{2\Lambda_{n+1}}\sin(\xi_{nl} - \Lambda_{n+1}t)\right.$$

$$\left. + \frac{\delta}{2\Lambda_{n-1}}\sin(\xi_{nl} - \Lambda_{n-1}t)\right],$$

$$u_z(r, t) = -\frac{q}{m}\left[\psi_1 k_\parallel + \frac{V_\perp k_\parallel k_\perp}{\omega}(\phi_1 - \psi_1)\frac{n_1}{\alpha_1}\right]A_1 A_2$$

$$\times \frac{1}{\Lambda_n}[\cos\xi_{nl} - \delta\cos(\xi_{nl} - \Lambda_n t)], \tag{6}$$

where $\delta$ 0 for the non-resonant particles and $\delta = 1$ for the resonant one and

$$A_1 = \sum_{n_1, n_4 = -\infty}^{\infty} J_{n1}(\alpha_1) J_{n2}(\alpha_2/2) J_{n3}(\alpha_2) J_{n4}(\alpha_2),$$

$$A_2 = \sum_{l_1 - l_4 = -\infty}^{\infty} J_{l1}(\alpha_1) J_{l2}(\alpha_2/2) J_{l3}(\alpha_2) J_{l4}(\alpha_2),$$

$$\alpha_1 = (k_\perp V_\perp/\Omega),\ \alpha_2 = (k_\perp \epsilon_B V_\perp^2/2\Omega^2),$$

$$\epsilon_B = \frac{1}{B}\frac{dB}{dx},$$

$$\Lambda_n = k_\parallel V_\parallel - \omega + (n_1 + 2n_2 - n_3 - n_4)\Omega,$$

$$a_n^2 = \Lambda_n^2 - \Omega^2,$$

$$\xi_{nl} = k_\perp x + k_\parallel z - \omega t + \{(n_1 - l_1) + 2(n_2 - l_2)$$

$$- (n_3 - l_3) - (n_4 - l_4)\}\Omega t - (n_1 - l_1)\theta$$

$$- \{(n_2 - l_2) - (n_3 - l_3)\}\cdot(\pi/2 + 2\theta)$$

$$+ (n_4 - l_4)\pi/2. \tag{7}$$

Also use was mode of the following.

$$\exp[-i\mu\sin(\theta - \Omega t)] = \sum_{-\infty}^{\infty} J_n(\mu)\exp[-in(\theta - \Omega t)],$$

$$\cos\theta\exp[-i\mu\sin\theta] = \frac{n}{\mu}\sum_{-\infty}^{\infty} J_n(\mu)\exp[-in\theta].$$

Integration of eq. (6) gives the perturbed coordinates of the particles $x$, $y$, $z$ which in addition to trajectories of free gyration, exhibits the true path of the particles. In view of the approximations introduced in the beginning, the dominant contribution comes from the term $n_1 = 0$. The resonant criterion is given by $k_\parallel V_\parallel - \omega = 0$. Therefore, this resonant condition means that the electrons see the wave independent of $t$ in the particles frame. The particles satisfying the above condition are called resonant. $J_s$ are Bessel's functions which arise from the different periodical variation of charged particles trajectories. The term represented by Bessel's functions shows the reduction of the field intensities due to finite gyroradius effect.

## 3. Density perturbation

In order to find out the density perturbation associated with the velocity perturbation, $\bar{u}(\bar{r}, t)$, we consider the equation [26,27,31]

$$\frac{dn_1}{dt} = -(\bar{\nabla}\cdot\bar{u})N - u_y\frac{dN}{dy}, \tag{8}$$

where $N(\bar{V})$ represents the zeroth-order distribution function. Expressing the right-hand-side of the eq. (8) as a function of $t$ [29] and after integration, we obtain the perturbed density for non-resonant and resonant perticles in the presence of the kinetic Alfvén wave for the inhomogeneous plasma as

$$n_1(\bar{r}, t) = N(\bar{V})A_1 A_2\frac{q}{m}\left[\left\{\phi_1 - \frac{V_\parallel k_\parallel}{\omega} \quad - \psi_1)\right.\right.$$

$$\times\left\{\frac{k_\perp^2}{a_n^2} + \frac{\Omega^2 V_d k_\perp m}{\Lambda_n a_n^2 T_\perp}\right\} + \frac{k_\parallel^2}{\Lambda_n^2}$$

$$\left.\times\left\{\psi_1 + \frac{n_1}{\alpha_1}\frac{V_\perp k_\perp}{\omega}(\phi_1 - \psi_1)\right\}\right]\cos\xi_{nl} \tag{9}$$

and for resonant particles

$$n_1(\bar{r}, t) = N(V)A_1 A_2\frac{q}{m}\left[\left\{\phi_1 - \frac{V_\parallel k_\parallel}{\omega}(\phi_1 - \psi_1)\right\}\right.$$

$$\left.\frac{k_\perp^2}{a_n^2} + \frac{\Omega^2 V_d k_\perp m}{\Lambda_n a_n^2 T_\perp}\right)\cos\xi_{nl} + \frac{1}{2\Omega\Lambda_{n+1}}$$

$$\times \cos(\xi_{nl} - \Lambda_{n+1}t)\left(k_\perp^2 - \frac{V_d k_\perp m\Omega}{T_\perp}\right) + \frac{V_d k_\perp m}{\Lambda_n T_\perp}$$

$$\times \cos(\xi_{nl} - \Lambda_n t) - \frac{1}{2\Omega\Lambda_{n-1}}\cos(\xi_{nl} - \Lambda_{n-1}t)$$

$$\times \left(k_\perp^2 + \frac{V_d k_\perp m\Omega}{T_\perp}\right)\Bigg\} + \frac{k_\parallel^2}{\Lambda_n^2}$$

$$\times \left\{\psi_1 + \frac{n_1}{\alpha_1}\frac{k_\perp V_\perp}{\omega}(\phi_1 - \psi_1)\right\}$$

$$\times \{\cos\xi_{nl} + \Lambda_n t \sin(\xi_{nl} - \Lambda_n t)$$

$$\cos(\xi_{nl} - \Lambda_n t)\}\Bigg] \tag{10}$$

where $V_d$ is the diamagnetic drift velocity which is defined by

$$V_d = \frac{T_\perp}{m\Omega}\frac{1}{N}\frac{\partial N}{\partial y}$$

and $V_d = 0$ represents the homogeneous plasma, $T_\perp$ is the perpendicular temperature.

To determine the dispersion relation and the growth rate, we consider a bi-maxwellian plasma with density distribution

$$N(y, \bar{V}) = N_0\left[1 - \epsilon\left(y + \frac{V_y}{\Omega}\right)\right] f_\perp(V_\perp) f_\parallel(V_\parallel),$$

$$\text{where } f_\perp(V_\perp) = \frac{m}{2\pi T_\perp}\exp(-mV_\perp^2/2T_\perp), \tag{11}$$

$$f_\parallel(V_\parallel) = \left(\frac{m}{2\pi T_\parallel}\right)^{1/2}\exp(-mV_\parallel^2/2T_\parallel)$$

where $\epsilon$ is a small parameter of the order of inverse of the density gradient scale length.

## 4. Dispersion relation

To evaluate the dispersion relation we calculate the integrated perturbed density for non-resonant particles as

$$\tilde{n}_{i,e} = \int_0^\infty 2\pi V_\perp dV_\perp \int_{-\infty}^{\infty} dV_\parallel n_{i,e}(\bar{r}, t). \tag{12}$$

With the help of eqs. (9) and (11), we find the average densities for inhomogeneous plasma as

$$= \frac{N_0 e}{m_i}\left[-\frac{k_\perp^2\phi}{\Omega_i^2} + \frac{k_\parallel^2\psi}{\omega^2} + \frac{V_d' k_\perp m_i}{T_{\perp i}\omega}\right. \tag{13a}$$

$$\text{where } B = \left(1 - \frac{1}{2}k_\perp^2\rho_i^2\right) - 9\left(\frac{k_\perp \epsilon_H T_{\perp i}}{2\Omega_i^2 m_i}\right)^2\left(1 - \frac{3}{2}k_\perp^2\rho_i^2\right.$$

$$\frac{\omega_{pe}^2}{4\pi e V_{T\parallel e}^2}\psi. \tag{13b}$$

It is observed that essential feature of the kinetic Alfvén wave is retained even in this ideal case. For Maxwell's equation, we use the quasi-neutrality condition [35]

$$\tilde{n}_i = \tilde{n}_e$$

and we get the relation between $\phi$ and $\psi$ as

$$\phi = -\frac{\Omega_i^2}{k_\perp^2}\left[\frac{\omega_{pe}^2}{\omega_{pi}^2 V_{T\parallel e}^2 B} - \frac{k_\parallel^2}{\omega^2}\right]\left(1 - \frac{v_d' k_\perp \Omega_i^2 m_i}{T_{\perp i} k_\perp^2 \omega}\right)^{-1}\psi. \tag{14}$$

Using perturbed ion and electron densities $\tilde{n}_i$ and $\tilde{n}_e$ and Ampere's law in the parallel direction [35], we obtain the equation

$$\frac{\partial}{\partial z}\nabla_\perp^2(\phi - \psi) = \frac{4\pi}{c^2}\frac{\partial}{\partial t}J_z, \tag{15}$$

$$\text{where } J_z = e\int_0^\infty 2\pi V_\perp dV_\perp \int_{-\infty}^\infty dV_\parallel\Big[\big(N(\bar{V})u_z(\bar{r}, t) + V_\parallel n_1(r, t)\big)_i$$

$$- \big(N(\bar{V})u_z(\bar{r}, t) + V_\parallel n_1(\bar{r}, t)\big)_e\Big],$$

$J_z$ is the current density which is contributed by first-order perturbations of density and velocity. With the help of eqs. (14) and (15), we obtain the dispersion relation for the kinetic Alfvén wave in inhomogeneous plasma as

$$\left(\frac{\omega^2}{k_\parallel^2 c_s^2 B}\right)\cdot\left(1 - \frac{\omega^2 B}{k_\parallel^2 v_A^2}D_d\right) = \frac{k_\perp^2\omega^2}{k_\parallel^2\Omega_i^2}D_d$$

$$\frac{\omega_{pi}^2\omega^2}{c^2\Omega_i^2 k_\parallel^2}\left(\frac{T_{\parallel i}}{m_i}\right)\left(\frac{\omega_{pe}^2}{\omega_{pi}^2 V_{T\parallel e}^2 B} - \frac{k_\parallel^2}{\omega^2}\right.$$

$$\left.\frac{v_d' k_\perp m_i}{T_{\perp i}\omega}D_d\right)\cdot B. \tag{16}$$

$$\text{where } D_d = \left(1 - \frac{v_d' k_\perp \Omega_i^2 m_i}{T_{\perp i} k_\perp^2 \omega}\right.$$

$$B = \left[\left(1 - \frac{1}{2}k_\perp^2\rho_i^2\right) - 9\left(\frac{k_\perp \epsilon_H T_{\perp i}}{2\Omega_i^2 m_i}\right)^2 \times \left(1 - \frac{3}{2}k_\perp^2\rho_i^2\right.\right.$$

$$\text{and} \quad c_s^2 = \frac{\omega_{pi}^2 V_{T\parallel e}^2}{\omega_{pe}^2},$$

is the square of ion-acoustic speed and

$$v_A^2 = \frac{c^2\Omega_i^2}{\omega_{pi}^2}$$

is the square of Alfvén speed. The dispersion relation of the kinetic Alfvén wave reduces to that derived by Hasegawa and Chen [3], Baronia and Tiwari [26], under the approximation $v_d' = 0$, $\epsilon_H = 0$ and $I_0(\lambda_i)\exp(-\lambda_i) \sim 1 - \lambda_i$, for $\lambda_i = \frac{1}{2}k_\perp^2\rho_i^2 < 1$ as we have applied. $I_0(\lambda_i)$ is the modified Bessel function.

## 5. Current density

Since the average value of current vanishes which is contributed by first-order perturbations of density and velocity due to their periodical variations, we evaluate the average current which is the second-order perturbation. To evaluate the perturbed current density we use the following set of equations

$$\bar{J}_{i,e} = \int_0^{\lambda} ds \int_0^{\infty} 2\pi V_\perp dV_\perp \int_{-\infty}^{\infty} dV_\parallel\, e \times \left[(N+n_1)(\bar{V}+\bar{u}) - N\bar{V}\right]_{i,e} \tag{17}$$

and $\bar{J} = \bar{J}_i - \bar{J}_e$.

With the help of eqs. (6), (9) and (11), we obtain

$$J_{xe} = \frac{N_0 e^3 k_\perp k_\parallel^2 \lambda}{2m_e^2 \Omega_e^2}\left[-\frac{\psi_1(\phi_1-\psi_1)}{\omega} - \frac{2\omega}{k_\parallel^2 V_{T\parallel e}^2}\psi_1^2\right] \times \left[1 - 18\left(\frac{k_\perp \epsilon_B T_{\perp e}}{2\Omega_e^2 m_e}\right)^2\right], \tag{18}$$

$$J_{xi} = \frac{N_0 e^3 k_\perp k_\parallel^2 \lambda}{2m_i^2 \Omega_i^2}\left[\frac{\phi_1-\psi_1}{\omega}\right] \times \left[(1-k_\perp^2\rho_i^2) - 18\left(\frac{k_\perp \epsilon_B T_{\perp i}}{2\Omega_i^2 m_i}\right)^2 (1-3k_\perp^2\rho_i^2)\right]. \tag{19}$$

Similarly, for the current in the $z$-direction

$$J_{ze} = \frac{N_0 e^3 \psi_1 k_\parallel \lambda}{2m_e^2}\left[\left\{-\frac{(\phi_1-\psi_1)}{\omega} - \frac{2\omega}{k_\parallel^2 V_{T\parallel e}^2}\psi_1\right\}\frac{k_\perp^2}{\Omega_e^2} - \frac{4\omega}{k_\parallel^2 V_{T\parallel e}^4}\psi_1\right] \times \left[1 - 18\left(\frac{k_\perp \epsilon_B T_{\perp e}}{2\Omega_e^2 m_e}\right)^2\right], \tag{20}$$

$$J_{zi} = \frac{N_0 e^3 k_\parallel \lambda \psi_1}{2m_i^2}\left[-\frac{k_\perp^2\phi_1}{\Omega_i^2\omega} - \frac{\psi_1}{\omega V_{T\parallel i}^2}\right] \times \left[(1-k_\perp^2\rho_i^2) - 18\left(\frac{k_\perp \epsilon_B T_{\perp i}}{2\Omega_i^2 m_i}\right)^2 (1-3k_\perp^2\rho_i^2)\right]. \tag{21}$$

Substituting eqs. (19) and (18) in eq. (17), we obtain

$$J_x = \frac{\psi_1 e k_\perp k_\parallel^2 \lambda}{8\pi}\left[\frac{\omega_{pe}^2}{m_e\Omega_e^2}\left\{\frac{(\phi_1-\psi_1)}{\omega} + \frac{2\omega}{k_\parallel^2 V_{T\parallel e}^2}\psi_1\right\} \times \left\{1 - 18\left(\frac{k_\perp \epsilon_B T_{\perp e}}{2\Omega_e^2 m_e}\right)^2\right\} - \frac{\omega_{pi}^2\phi_1}{m_i\Omega_i^2\omega}\left\{(1-k_\perp^2\rho_i^2) - 18\left(\frac{k_\perp \epsilon_B T_{\perp i}}{2\Omega_i^2 m_i}\right)^2 (1-3k_\perp^2\rho_i^2)\right\}\right] \tag{22}$$

$$J_z = \frac{e\psi_1 k_\parallel \lambda}{8\pi}\left[\frac{\omega_{pe}^2}{m_e}\left\{\left(\frac{(\phi_1-\psi_1)}{\omega} + \frac{2\omega}{k_\parallel^2 V_{T\parallel e}^2}\psi_1\right)\frac{k_\perp^2}{\Omega_e^2} + \frac{4\omega}{k_\parallel^2 V_{T\parallel e}^4}\psi_1\right\} \times \left\{1 - 18\left(\frac{k_\perp \epsilon_B T_{\perp e}}{2\Omega_e^2 m_e}\right)^2\right\} - \frac{\omega_{pi}^2}{m_i\omega}\left\{\frac{k_\perp^2\phi_1}{\Omega_i^2} + \frac{\psi_1}{V_{T\parallel i}^2}\right\} \times \left\{(1-k_\perp^2\rho_i^2) - 18\left(\frac{k_\perp \epsilon_B T_{\perp i}}{2\Omega_i^2 m_i}\right)^2 (1-3k_\perp^2\rho_i^2)\right\}\right]. \tag{23}$$

In the evaluation of the current densities, it was assumed that the field-aligned and perpendicular currents are due to an electromagnetic kinetic Alfvén wave and the contribution due to diamagnetic drift was neglected.

## 6. Energy balance and growth rate

The oscillatory motion of non-resonant electrons carries the major part of energy [29,31]. The wave energy density per unit wavelength $W_w$ is the sum of pure field energy and the changes in energy of the non-resonant particles $W_{i,e}$. It is observed that the wave energy is contained in the form of the oscillatory motion of the non-resonant electrons [29,31]. Thus

$$W_w = W_e = \frac{\lambda k_\parallel^2 \psi_1^2}{16\pi}\frac{\omega_{pe}^2}{k_\parallel^2 (T_{\parallel e}/m_e)}. \tag{24}$$

Now we calculate the resonance energy $W_r$ of the electrons per unit wavelength, that is

$$W_r = \int_0^{\lambda} ds \int_0^{\infty} 2\pi V_\perp dV_\perp \int_{(\omega/k_\parallel)-\Delta V_\parallel}^{(\omega/k_\parallel)+\Delta V_\parallel} \times \left(\frac{1}{2}Nm_e u_z^2 + n_1 m_e u_z V_\parallel\right) dV_\parallel. \tag{25}$$

With the help of eqs. (6), (10), (11) and (25) expanding the integrand around $V_\parallel = \omega/k_\parallel$ and following the procedure as discussed in Tiwari and Varma [29] and Terashima [31] in the limiting case of $k_\perp\rho_e << 1$, we obtain

$$W_r = \pi^{1/2}\frac{\lambda k_\parallel^2 \psi_1^2}{8\pi}\frac{\omega_{pe}^2 \omega t}{k_\parallel^2 (T_{\parallel e}/m_e)}\left[1 - 18\left(\frac{k_\perp \epsilon_B T_{\perp e}}{2\Omega_e^2 m_e}\right)^2\right] \times \left[\frac{\omega - k_\perp v_d^e (T_{\parallel e}/T_{\perp e})}{k_\parallel (2T_{\parallel e}/m_e)^{1/2}}\right] \times \exp\left(-\frac{m_e\omega^2}{2T_{\parallel e}k_\parallel^2}\right), \tag{26}$$

where $ks = \bar{k}.\bar{r}$ and $\bar{k} = 2\pi/\lambda$ and $\omega_{pi,e}^2 = 4\pi N_0 e^2/m_{i,e}$.

Using the law of conservation of energy, we calculate the growth rate of the drift kinetic Alfvén wave by

$$\frac{d}{dt}(W_w + W_r) = 0. \tag{27}$$

With the help of (24) and (26), we have found the growth rate of the drift kinetic Alfvén wave as

$$\gamma/\omega = \pi^{1/2}\frac{\omega}{k_{\parallel}V_{T\parallel e}}\left[\frac{T_{\parallel e}k_{\perp}v_d^e}{T_{\perp e}\omega}-1\right]$$

$$\times\left[1-18\left(\frac{k_{\perp}\in_B T_{\perp e}}{2\Omega_e^2 m_e}\right)^2\right]\times\exp\left(-\frac{\omega^2}{k_{\parallel}^2V_{T\parallel e}^2}\right). \quad (28)$$

where $V_{T\parallel}^2 = (2T_{\perp,\parallel}/m)$; $v_d^e$ represents electron diamagnetic drift velocity and the value of $\omega$ for the drift kinetic Alfvén wave has to be substituted. In the case $\in_B = 0$, we recover the growth rate as derived by Baronia and Tiwari [26,27].

It is noted that the kinetic Alfvén wave can be excited only when $(T_{\parallel e}/T_{\perp e})k_{\perp}v_d^e > \omega$. Thus, the kinetic Alfvén wave is excited as the usual drift wave.

## 7. Results and discussion

In the numerical evaluation of the growth rate, current and dispersion relation, we have used the following parameters for the auroral acceleration region [26,27,36]

$B_0 = 4300\ nT$, $\Omega_i = 412\ \mathrm{s}^{-1}$, $kT_{\parallel i} = 1$ keV, $kT_{\parallel e} = 10$ keV, $v_d^e = 200$ cm/sec, $\omega_{pi}^2/\Omega_i^2 = 100$.

Figure 1 shows the variation of wave frequency $\omega$ (rad/sec) *versus* perpendicular wave number $k_{\perp}\rho_i$ for different

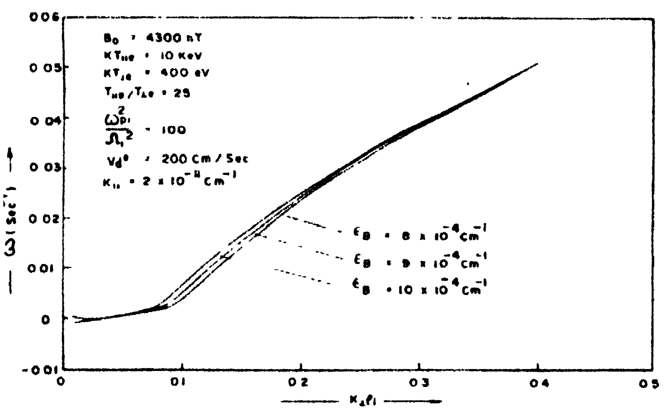


**Figure 1.** Frequency ($\omega$) *versus* perpendicular wave number ($k_{\perp}\rho_i$) for different $\in_B$

values of magnetic field gradient $\in_B$. It is noticed that wave frequency $\omega$ increases with $k_{\perp}\rho_i$ and that the magnetic field gradient $\in_B$ is effective for limited range of $k_{\perp}\rho_i$ as the $\nabla B$ drift velocity is smaller than the diamagnetic drift velocity. Therefore, for low-$\beta$ plasma $\beta << 1$, $\nabla B$ drift is usually, less effective and the effects of magnetic inhomogeneities are small in dispersion relation derived in eq. (16). This would justify the low $\beta$ approximation discussed in our analysis. The small change in the wave frequency $\omega$ is due to magnetic drift of the charge particles. Figure 2 shows the variation of growth rate $\gamma/\omega$ *versus* $k_{\perp}\rho_i$ for different values of $\in_B$. It is seen that a higher magnetic gradient reduces the growth rate and permits a higher frequency for emission. For higher $k_{\perp}\rho_i$, there exists no frequency band. The reduction in growth rate due to $\in_B$ towards lower $k_{\perp}\rho_i$ is also clear from the figure. The increase of $\in_B$ narrows down the emission band and the wave can be generated for the limited range of $k_{\perp}\rho_i$. The stabilizing effect of magnetic field inhomogeneity is also clear as the growth rate reduces for all values of $k_{\perp}\rho_i$. This may be due to the weakening of wave electric field due to Larmor radius effect.

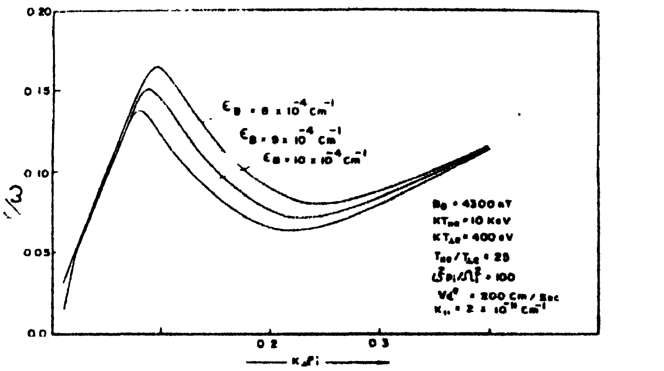


**Figure 2.** Growth rate ($\gamma/\omega$) *versus* perpendicular wave number ($k_{\perp}\rho_i$) for different $\in_B$.

Figure 3 and 4 show the variation of currents *versus* $k_{\perp}\rho_i$ for different values of $\in_B$ in the perpendicular and parallel

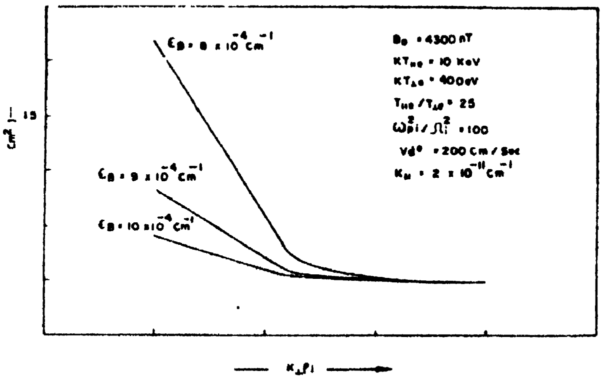


**Figure 3.** Perpendicular current ($J_x$) *versus* perpendicular wave number ($k_{\perp}\rho_i$) for different $\in_B$.

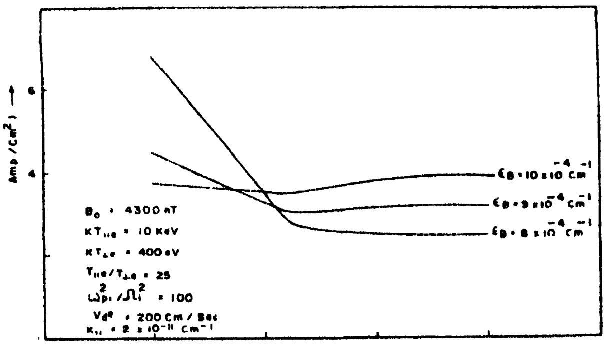


**Figure 4.** Parallel current ($J_z$) *versus* perpendicular wave number ($k_{\perp}\rho_i$) for different $\in_B$

directions. It is seen that $J_x$ and $J_z$ decrease with the increase of $k_\perp\rho_i$ as well as $\epsilon_B$. Both the figures predict that current can be generated by the drift kinetic Alfvén wave and constitute coupled system of perpendicular and parallel currents-as well as the potential drop along the auroral field lines in the acceleration region. The critical study of energy of accelerated particles along auroral field lines and associated phenomena is the matter of further investigation. Figures 3 and 4 also predict that both the currents $J_x$ and $J_z$ are higher at the lower values of $k_\perp\rho_i$. However, $J_x$ decreases at higher $k_\perp\rho_i$ due to weakening of perpendicular component of wave electric field because of Larmor radius effect and $J_z$ increases to maintain the current continuity. The increase in $J_z$ at higher $k_\perp\rho_i$ with magnetic inhomogeneity may be due to accelerated particles as the growth rate becomes smaller in this band.

Figure 5 shows the variation of change in normalized particle energy density ($W_r/W_w$) *versus* the $k_\perp\rho_i$ for different

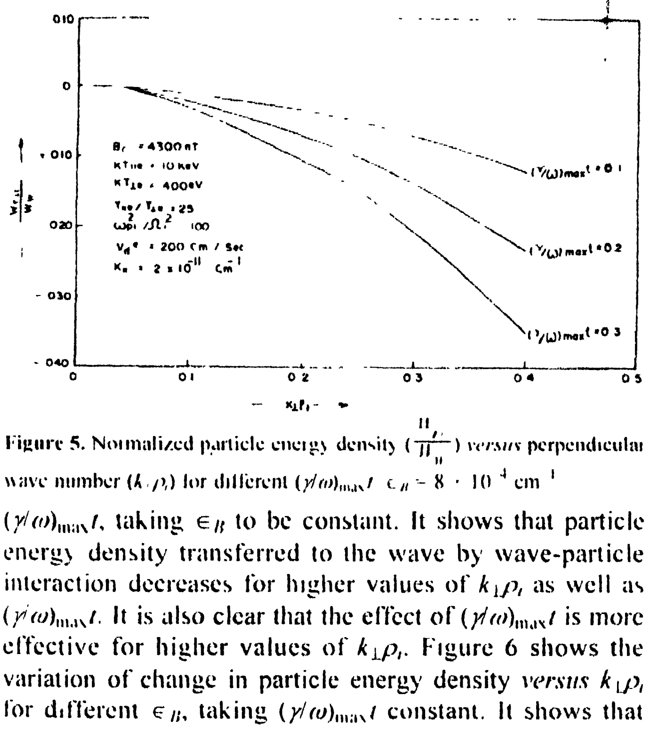

Figure 5. Normalized particle energy density ($\frac{W_r}{W_w}$) *versus* perpendicular wave number ($k_\perp\rho_i$) for different $(\gamma/\omega)_{max}t$, $\epsilon_B = 8 \times 10^{-4}$ cm$^{-1}$

$(\gamma/\omega)_{max}t$, taking $\epsilon_B$ to be constant. It shows that particle energy density transferred to the wave by wave-particle interaction decreases for higher values of $k_\perp\rho_i$ as well as $(\gamma/\omega)_{max}t$. It is also clear that the effect of $(\gamma/\omega)_{max}t$ is more effective for higher values of $k_\perp\rho_i$. Figure 6 shows the variation of change in particle energy density *versus* $k_\perp\rho_i$ for different $\epsilon_B$, taking $(\gamma/\omega)_{max}t$ constant. It shows that change in particle energy density increases with $\epsilon_B$ for higher values of $k_\perp\rho_i$. Thus, the energy transferred to the wave in inhomogeneous magnetic field is higher and predicts an impulsive phenomena, an unique feature of auroral acceleration process. Energetic particles may create a temperature anisotropy at the substorm times which may also be the cause of the drift kinetic Alfvén wave emissions [26].

The findings of the investigation may be useful for the plasma heating processes, confinement device and the space plasmas. The KAW is fundamentally important for supplementary heating of a Tokamak type plasma [24]. With the help of this study, one can explain the formation of the auroral arcs and the associated phenomena at the substorm times.

The field-aligned currents are the critical feature of magnetosphere-ionosphere coupling. In this work, we have intended to attribute their origin to plasma density inhomogeneity of the magnetosphere. The sub-auroral region

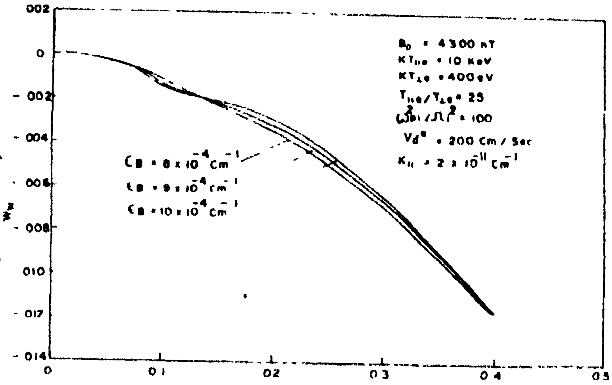

Figure 6. Normalized particle energy density ($\frac{W_{r\parallel}}{W_w}$) *versus* perpendicular wave number ($k_\perp\rho_i$) for different $\epsilon_B$ $(\gamma/\omega)_{max}t = 0.1$

-II field aligned current system is commonly attributed to pressure driven current sources [20]. When change in the magnetospheric pressure take place suddenly (for example, when a magnetospheric substorm expansion take place), the plasma density changes will lead to a new field-aligned current distribution which will transiently be carried out to the ionosphere along the field by kinetic Alfvén wave that will bounce back and forth on closed field lines. Well-known pi-2 geomagnetic pulsations commonly associated with the onset of substorms are the signatures of the pressure driven changes of the field-aligned current system.

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